Two's Complement Math

Two's complement is a method for representing signed integers in binary, allowing both positive and negative numbers to be expressed. It's widely used in computer systems for arithmetic operations because it simplifies the hardware design for addition and subtraction.

How Two's Complement Works

In an n-bit binary system, the leftmost bit (most significant bit) is used as the sign bit:
- 0 indicates a positive number.
- 1 indicates a negative number.

Steps to Find the Two's Complement of a Number

1. Write the binary representation of the absolute value of the number.

2. Invert all bits (change 0s to 1s and 1s to 0s).

3. Add 1 to the inverted binary number.

This result represents the negative of the original absolute value in two's complement form.

Examples

Example 1: Converting -6 to Two's Complement (8-bit representation)

1. Start with the positive binary of 6:
   00000110

2. Invert all bits:
   11111001

3. Add 1 to the inverted result:
   11111001 + 1 = 11111010

So, -6 is represented as 11111010 in two's complement (8-bit).

Example 2: Converting -1 to Two's Complement (8-bit representation)

1. Binary of +1:
   00000001

2. Invert all bits:
   11111110

3. Add 1:
   11111110 + 1 = 11111111

Thus, -1 is 11111111 in two's complement.

Addition and Subtraction Using Two's Complement

The beauty of two's complement is that you can perform subtraction by adding the negative (two's complement) version of a number. This simplifies calculations, as addition is handled uniformly regardless of the sign.

Example: 5 + (-3) Using Two's Complement (8-bit)

1. Binary for +5:
   00000101

2. Binary for +3:
   00000011

   Find the two's complement of +3 to represent -3:
   - Invert: 11111100
   - Add 1: 11111101

3. Add the two numbers:
   00000101 + 11111101 = 100000010

Since this result is in 9 bits, drop the leftmost 1 to get 00000010, which is the binary for +2. Therefore, 5 + (-3) = 2.

Key Points of Two's Complement

- It provides a seamless way to perform arithmetic operations with signed numbers.
- The range for an n-bit system is -2^(n-1) to 2^(n-1) - 1.
- The leftmost bit serves as the sign bit, making the interpretation straightforward for both positive and negative values.

Two's complement is efficient and standard in computer systems, making arithmetic with signed integers simpler and more consistent.